Complex seismic trace analysis

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The conventional seismic trace can be viewed as the real component of a complex trace which can be uniquely calculated under usual conditions. The complex trace permits the unique separation of envelope amplitude and phase information and the calculation of instantaneous frequency. These and other quantities can be displayed in a color-encoded manner which helps an interpreter see their interrelationship and spatial changes. The significance of color patterns and their geological interpretation is illustrated by examples of seismic data from three areas.

INTRODUCTION

This paper has two objectives: specifically to (1) explain the application of complex trace analysis to seismic data and its usefulness in geologic interpretation and (2) illustrate the role of color in conveying seismic information to an interpreter. Expressing seismic data in complex form also yields computational advantages which are discussed in Appendix A.

Transformations of data from one form to another are common in signal analysis, and various techniques are used to extract significant information from time series (seismic data). Interpreting data from different points of view often results in new insight and the discovery of relationships not otherwise evident.

The transformation of seismic data from the time domain to the frequency domain is the most common example of data rearrangement which provides insight and is useful in data analysis. The Fourier transform, which accomplishes this, allows us to look at average properties of a reasonably large portion of a trace, but it does not permit examination of local variations. Analysis of seismic data as an analytic signal, complex trace analysis, is a transform technique which retains local significance. Complex trace analysis provides new insight, like Fourier transforms, and is useful in interpretation problems.

Complex trace analysis effects a natural separation of amplitude and phase information, two of the quantities (called "attributes") which are measured in complex trace analysis. The amplitude attribute is called "reflection strength". The phase information is both an attribute in its own right and the basis for instantaneous frequency measurement. Amplitude and phase information are also combined in additional attributes, weighted average frequency and apparent polarity.

Signal analysis can also be viewed as a communications problem. The objective is to make an interpreter aware of the information content of data, including an appreciation for the reliability of measurements and how information elements relate to each other. The display of data is an inherent part of the analysis. Seismic data are conventionally displayed in variable area, variable density, variable amplitude (wiggle), or a combination of these forms. Display scale and vertical-to-horizontal scale ratio are variables whose judicious choice can aid analysis (Sheriff and Farrell, 1976). Display parameters also include trace superposition, bias, and color. Color has proven to be especially effective in complex trace analysis.

The literature on the use of color in geophysics is limited. Balch (1971) discussed the use of color seismic sections as an interpretation aid, and geological advertisements have illustrated limited use.

Presented at the 46th Annual International SEG Meeting October 27, 1976 in Houston and at the 47th Annual International SEG Meeting September 21, 1977 in Calgary. The subject matter constituted the lectures given by M. T. Taner as AAPG Distinguished Lecturer in 1975 and by R. E. Sheriff as SEG Distinguished Lecturer in 1977. Manuscript received by the Editor January 23, 1978; revised manuscript received August 7, 1978.

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CALCULATION OF THE COMPLEX TRACE

Complex trace analysis is discussed in electrical engineering and signal analysis literature (Gabor, 1946; Bracewell, 1965; Cramer and Leadbetter, 1967; Oppenheim and Schafer, 1975). Some applications to seismic signal problems are given in Farnbach (1976) and Taner and Sheriff (1977). However, explanation of the application to seismic signal analysis is not available in the geophysical literature.

Basic definitions

Complex trace analysis treats a seismic trace \( f(t) \) as the real part of an analytical signal or complex trace, \( F(t) = f(t) + j f^*(t) \). The quadrature (also called conjugate or imaginary) component \( f^*(t) \) is uniquely determinable from \( f(t) \) if we require that

1) be determined from \( f(t) \) by a linear convolution operation, and
2) reduces to phasor representation if \( f(t) \) is a sinusoid, that is, \( f^*(t) = A \sin(\omega t + \theta) \) if \( f(t) = A \cos(\omega t + \theta) \) for all real values of \( A \) and \( \theta \) and all \( \omega > 0 \).

These rules determine \( f^*(t) \) uniquely for any function \( f(t) \) which can be represented by a Fourier series or Fourier integral.

The use of the complex trace \( F(t) \) makes it possible to define instantaneous amplitude, phase, and frequency in ways which are logical extensions of the definitions of these terms for simple harmonic oscillation. Complex traces can also be used in similarity calculations, enabling us to find more precisely the relative arrival times of a common signal appearing on different traces (Appendix A).

The real seismic trace \( f(t) \) can be expressed in terms of a time-dependent amplitude \( A(t) \) and a time-dependent phase \( \theta(t) \) as

\[
f(t) = A(t) \cos \theta(t).
\]

The quadrature trace \( f^*(t) \) then is

\[
f^*(t) = A(t) \sin \theta(t),
\]

and the complex trace \( F(t) \) is

\[
F(t) = f(t) + j f^*(t) = A(t) e^{j \theta(t)}.
\]

If \( f(t) \) and \( f^*(t) \) are known, one can solve for \( A(t) \) and \( \theta(t) \):

\[
A(t) = \left[ f^2(t) + f^*2(t) \right]^{1/2} = |F(t)|,
\]

and

\[
\theta(t) = \tan^{-1} \left[ f^*(t) / f(t) \right].
\]

\( A(t) \) is called “reflection strength,” and \( \theta(t) \) is called “instantaneous phase” (Bracewell, 1965).

The rate of change of the time-dependent phase gives a time-dependent frequency

\[
\frac{d \theta(t)}{dt} = \omega(t).
\]

This can be expressed in convolutional form as

\[
\omega(t) = \int_{-\infty}^{\infty} d(\tau) \theta(t - \tau) d\tau,
\]

where \( d(\tau) \) is the differentiation filter (Rabiner and Gold, 1975, p. 164). A difficulty with this is that the phase must be continuous, whereas the arctangent computation of equation (5) gives only the principle value. We then have to “unwind” the phase by determining the location of \( 2\pi \) phase jumps and correcting them.

A more convenient way of computing the instantaneous frequency is to compute the derivative of
the arctangent function itself

\[ \omega(t) = \frac{d}{dt} \left\{ \tan^{-1} \left[ \frac{f^*(t)}{f(t)} \right] \right\}, \]  

which results in

\[ \omega(t) = \frac{f(t) \frac{df^*(t)}{dt} - f^*(t) \frac{df(t)}{dt}}{f^2(t) + f^*2(t)}, \]  

where the derivatives of \( f(t) \) and \( f^*(t) \) can be computed in convolutional form as in equation (7).

We also define a weighted average frequency \( \bar{\omega}(t) \) as

\[ \bar{\omega}(t) = \frac{\int_{-\infty}^{\infty} A(t - \tau) \omega(t - \tau) L(\tau) d\tau}{\int_{-\infty}^{\infty} A(t - \tau) L(\tau) d\tau}, \]  

where \( L(\tau) \) is a low-pass filter. Apparent polarity is defined as the sign of \( f(t) \) when \( A(t) \) has a local maximum. Positive or negative sign is assigned assuming a zero-phase wavelet and a positive or negative reflection coefficient, respectively.

**Calculation of the quadrature trace**

We give equivalent ways of defining \( f^*(t) \) and \( F(t) \), first in terms of Fourier integrals and then by convolution in the time domain using the Hilbert transform.

We assume that \( f(t) \) is real, defined for \(-\infty < t < \infty\), and can be represented by the Fourier integral formula

\[ f(t) = \int_{-\infty}^{\infty} B(\omega) e^{i\omega t} d\omega, \]  

and

\[ f(t) = \int_{0}^{\infty} C(\omega) \cos [\omega t + \phi(\omega)] d\omega, \]  

where \( C(\omega) = 2|B(\omega)| \) and \( \phi(\omega) = \arg B(\omega), \omega > 0 \). Then

\[ f^*(t) = \int_{0}^{\infty} C(\omega) \sin [\omega t + \phi(\omega)] d\omega; \]  

and

\[ F(t) = \int_{0}^{\infty} C(\omega) e^{i(\omega t + \phi(\omega))} d\omega. \]  

The frequency-domain representations of a real trace and its complex trace equivalent are shown in Figure 1. The amplitude spectrum of the complex trace \( C(\omega) \) vanishes for \( \omega < 0 \) and has twice the magnitude for \( \omega > 0 \). The phase \( \phi(\omega) \) is unchanged (except it is not defined for \( \omega < 0 \)). The complex trace can thus be found by (1) Fourier transforming the real trace, (2) zeroing the amplitude for negative frequencies and doubling the amplitude for positive frequencies, and then (3) inverse Fourier transforming.

An equivalent formula for \( f^*(t) \) is given by the Hilbert transform (Rabiner and Gold, 1975)

\[ f^*(t) = \frac{1}{\pi} \text{P.V.} \int_{-\infty}^{\infty} \frac{f(t) - f(t - \tau)}{t - \tau} d\tau, \]  

where P.V. \( f^\infty \) means the Cauchy principal value

\[ \text{P.V.} \int_{-\infty}^{\infty} = \lim_{\varepsilon \to 0} \left[ \int_{-\infty}^{t-\varepsilon} + \int_{t+\varepsilon}^{\infty} \right]. \]  

The Hilbert transform can be used to generate the quadrature trace from the real trace or vice versa by the convolution operation, which in digital form is

\[ f^*(t) = \frac{1}{\pi} \sum_{n=-\infty}^{\infty} f(t - n\Delta t) \frac{1 - e^{i\pi n}}{n}, \]  

and

\[ f^*(t) = \frac{2}{\pi} \sum_{n=-\infty}^{\infty} f(t - n\Delta t) \frac{\sin^2(\pi n/2)}{n}, \]  

for \( n \neq 0 \).
Fig. 3. Real (a) and quadrature (b) traces for a portion of an actual seismic trace. Also shown is the envelope [dotted curve in (a, b)], phase (c), instantaneous frequency (d), and weighted average frequency [dotted curve in (d)].
where $\Delta t$ is the sample interval. The inverse convolution is merely the negative

$$f(t) = -\frac{2}{\pi} \sum_{n=-\infty}^{\infty} f^*(t - n\Delta t) \frac{\sin^2(\pi n/2)}{n}, \ n \neq 0.$$  

The normalized Hilbert time-domain operator [equation (15)], shown in Figure 2, is odd, vanishes for even $n$, and decreases monotonically in magnitude as $|n|$ increases for odd $n$. It is usually applied in a modified truncated version.

**Graphical representations and examples**

The real $f(t)$ and quadrature $f^*(t)$ traces can be plotted in any of the conventional ways used for seismic traces. Variable amplitude plots for a portion of an actual seismic trace are shown in Figures 3a and 3b for the real and quadrature traces. The complex trace $F(t)$ can be thought of as the trace in complex space of a vector which is continually changing its length and rotating, thus tracing out an irregular helix as shown in Figure 4. We may then think of $A(t)$ as the time-varying modulus and $\theta(t)$ as the time-varying argument of this vector.

The seismic trace shown in Figures 3 and 4 is from an East Texas survey. The real and quadrature traces are given by the projection of the trace of the rotating vector on the real and imaginary planes, as shown in Figure 4. The length of the vector is $A(t)$ and its angle with the horizontal is $\theta(t)$.

Figure 5a shows a simple Ricker wavelet $f(t)$ and the quadrature trace $f^*(t)$ derived from it. Also shown is the magnitude $|F(t)| = A(t)$ and the phase $\theta(t)$. Figure 5b is an isometric diagram of the same wavelet showing the quadrature component $f^*(t)$ in the imaginary plane perpendicular to the real component $f(t)$. Figure 5c is a polar plot of $A(t) = |F(t)|$ at successive and equal time intervals, and Figure 5d is the corresponding amplitude spectrum $A(\omega)$. Data for this example are tabulated in Table 1 of Appendix B.

Note in Figures 3a and 3b that both real and quadrature traces are identical except phase shifted by 90 degrees. Except for this phase shift, a geophysicist would observe the same features, that is, the same
FIG. 5. (a) Real part $f(t)$, quadrature part $f^*(t)$, complex amplitude $|F(t)|$, and phase $\theta(t)$ of 25-Hz Ricker wavelet. (b) Isometric diagram of real, $f(t)$, and quadrature, $f^*(t)$, components of 25-Hz Ricker wavelet. (c) Polar plot of $A(t) = |F(t)|$ for a 25-Hz Ricker wavelet. (d) Spectrum $B(\omega)$ of 25-Hz Ricker wavelet.
coherency and the same signal-to-noise ratio, on real
and quadrature seismic sections.

The reflection strength $A(t)$ is the envelope of
the seismic trace. We might imagine the reflection
strength rotated about the time axis so as to appear like
beads on a string, sometimes overlapping, each bead
representing the arrival of new energy. The vector
rotates within each of these beads and the phase
(Figure 3c) occasionally has to back up or hurry ahead
to represent succeeding energy. The instantaneous
frequency curve (Figure 3d) jumps sharply whenever
the rotating vector is locking onto new energy but
does not change appreciably during each bead of
energy.

An average of the instantaneous frequency, such
as given by the weighted average frequency, yields
roughly the same value we would obtain if we were
to measure the period between successive points of
similar phase for succeeding cycles, as is often done
to determine “dominant” frequency. For the 25-Hz
Ricker wavelet of Figure 5,

$$\bar{\omega} = 24.5 \text{ Hz},$$

which is close to the 25-Hz value of $\omega$ at the peak of
its amplitude spectrum (Figure 5d).

**SIGNIFICANCE OF ATTRIBUTES**

Attribute measurements based on complex trace
analysis were defined in the preceding section. We
now examine their significance and color representa-
tions as originally described by Taner et al (1976).

**Reflection strength**

Reflection strength (amplitude of the envelope) is
defined by equation (4). Reflection strength is in-
dependent of phase. It may have its maximum at phase
points other than peaks or troughs of the real trace,
especially where an event is the composite of several
reflections. Thus, the maximum reflection strength
associated with a reflection event may be different
from the amplitude of the largest real-trace peak or
trough.

High-reflection strength is often associated with
major lithologic changes between adjacent rock
layers, such as across unconformities and boundaries
associated with sharp changes in sea level or de-
positional environments. High-reflection strength also
is often associated with gas accumulations. Strength
of reflections from unconformities may vary as the
subcropping beds change, and reflection strength
measurement may aid in the lithologic identification
of subcropping beds if it can be assumed that de-
position is constant above the unconformity so that
all the change can be attributed to subcropping beds.

Lateral variations in bed thicknesses change the
interference of reflections; such changes usually occur
over appreciable distance and so produce gradual
lateral changes in reflection strength. Sharp local
changes may indicate faulting or hydrocarbon accumulations where trapping conditions are Favor-
able. Hydrocarbon accumulations, especially gas,
may show as high-amplitude reflections or “bright
spots”. However, such bright spots may be non-
commercial and, conversely, some gas productive
zones may not have associated bright spots.

Observing where, within a reflection event, the
maximum reflection strength occurs provides a mea-
sure of reflection character. Occasionally, this can
be used to indicate reflection coefficient polarity
as shown by Taner and Sheriff (1977, p. 327).
Constancy of character may aid in distinguishing
between reflection events from a single reflector and
those which are a composite of reflections. The
strength of reflections from the top of massive beds
tends to remain constant over a large region. Ref-
lections of nearly constant strength provide good
references for time-interval measurements.

The usual color-encoding of reflection strength is
referred to the maximum reflection strength which
occurs on a seismic section or in an area, using a
different color for each dB step (Figure 6a). Using
the same color reference for the data over an area
provides color ties at line intersections, providing
data recording conditions were uniform or corrections
for nonuniform recording conditions were made in
processing. The reference can be changed where
desired.

**Instantaneous phase**

The instantaneous phase, defined by equation (5),
emphasizes the continuity of events. Instantaneous
phase is a value associated with a point in time and
thus is quite different from phase as a function of
frequency, such as given by the Fourier transform.
In phase displays, the phase corresponding to each
peak, trough, zero-crossing, etc. of the real trace is
assigned the same color so that any phase angle can
be followed from trace to trace.

Because phase is independent of reflection strength,
it often makes weak coherent events clearer. Phase
displays are effective in showing discontinuities,
faults, pinchouts, angularities, and events with differ-
ent dip attitudes which interfere with each other.
Prograding sedimentary layer patterns and regions of on-lap and off-lap layering often show with special clarity so that phase displays are helpful in picking "seismic sequence boundaries" (Payton, 1977, p. 310).

Phase displays use the colors of the color wheel (Figure 6b) so that plus and minus 180 degrees are the same color (purple) because they are the same phase angle. The cosine of the instantaneous phase angle is also displayed in black and white and is often used as a background for other displays (as in Figures 8 and 10-13).

**Instantaneous frequency**

Instantaneous frequency, defined by equation (6), is a value associated with a point in time, like instantaneous phase. Most reflection events are the composite of individual reflections from a number of closely spaced reflectors which remain nearly constant in acoustic impedance contrast and separation. The superposition of individual reflections may produce a frequency pattern which characterizes the composite reflection. Frequency character often provides a useful correlation tool. The character of a composite reflection will change gradually as the sequence of layers gradually changes in thickness or lithology. Variations, as at pinchouts and the edges of hydrocarbon-water interfaces, tend to change the instantaneous frequency more rapidly.

A shift toward lower frequencies ("low-frequency shadow") is often observed on reflections from reflectors below gas sands, condensate, and oil reservoirs. Low-frequency shadows often occur only on reflections from reflectors immediately below the petrolierous zone, reflections from deeper reflectors appearing normal. This observation is empirical and many have made the same observation, but we do not understand the mechanism involved. Two types of explanations have been proposed: (1) that a gas sand actually filters out higher frequencies because of (a) frequency-dependent absorption or (b) natural resonance, or (2) that traveltime through the gas sand is increased by lower velocity such that reflections from reflectors immediately underneath are not summed properly. These explanations seem inadequate to account for the observations. Fracture zones in brittle rocks are also sometimes associated with low-frequency shadows.

Frequency is usually color-coded in 2-Hz steps (Figure 6c). The red-orange end of the spectrum usually indicates the lower frequencies and the blue-green end, the higher frequencies. Frequencies lower than 6 Hz are usually left uncolored.

**Weighted average frequency**

Weighted average frequency, defined by equation (10), emphasizes the frequency of the stronger reflection events and smooths irregularities caused by noise. The frequency values approximate dominant frequency values determined by measuring peak-to-peak times or times between other similar phase points. Like instantaneous frequency displays, weighted average frequency displays are sometimes excellent for enhancing reflection continuity. Hydrocarbon accumulations often are evidenced by low frequencies.

**Apparent polarity**

While all attribute measurements depend on data quality and the quality of the recording and processing, apparent polarity measurements are especially sensitive to data quality. Interference may result in the reflection strength maximum occurring near a zero-crossing of the seismic trace so that the polarity may change sign as noise causes the zero-crossing of the trace or the location of the reflection strength maximum to shift slightly. The analysis of apparent polarity assumes a single reflector, a zero-phase wavelet, and no ambiguity due to phase inversion. However, since most reflection events are composites of several reflections, polarity often lacks a clear correlation with reflection coefficient and hence it is qualified as apparent polarity.

Polarity sometimes distinguishes between different kinds of bright spots (Figures 7c and 7f). Bright spots associated with gas accumulations in elastic sediments usually have lower acoustic impedance than surrounding beds and hence show negative polarity for reservoir top reflections and positive polarity for reflections from gas-oil or gas-water interfaces (often called "flat spots") (Figure 8c, event D).

Ordinarily, apparent polarity is color-coded magenta and blue for positive and negative, respectively, with the hue intensity graded in five steps according to reflection strength (Figure 6d).

**Display of attributes**

Each attribute to be displayed involves a value associated with each sample point. Assimilating and digesting such masses of data pose a major problem. Our usual practice is to color-encode the data and display these in a seismic-section format most
Figure 6. Color codes for attribute values. (a) Reflection strength; (b) phase; (c) frequency; (d) polarity.
Figure 7. Two portions (left and right) of a seismic section for Gulf of Mexico line A. Top: reflection strength; center: instantaneous frequency; bottom: apparent polarity.
Figure 8. Portion of seismic section for Southern North Sea line A. Top: instantaneous phase; center: weighted average frequency; bottom: apparent polarity.
Figure 13. Seismic section for Gulf of Mexico line C. Above: reflection strength; below: weighted average frequency.
familiar to interpreters, that is, location along a seismic line as abscissa and reflection time as ordinate. Color-encoded attribute measurements are often superimposed on a conventional seismic section so that one can see both the conventional data and the color-encoded attribute simultaneously, thus making it easier to see interrelations. The color-encoding involves assigning a color to each value or range of values. This assignment can be arbitrary, but assigning colors in spectral sequence is most natural in making relative magnitude clear.

The examples in this article were produced by the Seis-chrome® process which produces exactly the same color whenever the same value occurs. Many distinctly different colors can be used. A color code (Figure 6) is usually provided so that one can determine the numerical value associated with any sample location and thus interpret color changes quantitatively.

Displays which have been used include
1) compressed time scale ("squash plot") and normal horizontal scale, and enlargement ("zooms") of zones of special interest;
2) conventional traces or instantaneous phase as a black-and-white background in variable area, density, or amplitude modes; and
3) blanking out of samples at zero-crossing of the conventional trace so as to produce white lines every half cycle to indicate structure.

Processing of data prior to display often involves
1) phase filtering to convert to a nearly zero-phase wavelet;
2) predetermined time-dependent gain to accentuate the color display for the zone of interest; and
3) migration prior to analysis so that seismic events more nearly conform to positions of related subsurface features.

General interpretation considerations

The various attributes reveal more as a set than they do individually. Features often are anomalous in systematic ways on various displays. As an example of the value of multiple displays, Figure 7 shows two portions (left and right) of a seismic section for Gulf of Mexico line A. (This section is shown in Taner and Sheriff, 1977, Figures 8-12.) The reflection-strength displays (a and d) show high-amplitude events (bright spots), as indicated by the red and orange colors. The bright spots (red) on the left display (a) indicate a gas reservoir, though the reflection at 0.650 sec is associated with a non-commercial gas-reservoir. The bright spot on the right display (d) is associated with a local deposit of shells. The gas-reservoir zones have low-frequency shadows immediately underneath them, as shown by the yellow-orange in the frequency display (b), whereas the shell deposit produces no systematic change in frequency (e). The polarity of the gas-reservoir reflections, as seen in (e), is negative (blue color) and that of the shell deposit (f) is positive (magenta color). Thus the ensemble of displays makes clear that the two bright spots represent different subsurface features.

Most stratigraphic interpretation begins with the interpretation of structure (Sheriff, 1976). Attribute patterns aid in correlation, and the offset of patterns helps establish throw across faults. However, it is variation along the bedding that is of principle interest in attribute interpretation. Lateral variation in pattern suggests stratigraphic or other changes. Sometimes the meaning of a variation is clear, but often the meaning is clear only when well data are correlated to seismic data (Sheriff et al, 1977). As more additional data are assimilated, the more interpretable are these attribute measurements. Those familiar with local geology find significance which others miss. The use of lateral changes of pattern, especially low-frequency zones, to define limits of production is obviously important.

Compressing the horizontal scale allows a greater length of seismic section to be comprehended. The vertical exaggeration which results is often helpful in delineating stratigraphic changes which occur over long distances. Such a section does distort structure, however, causing faults to appear steeper, etc.

Attribute interpretation can be made on data migrated so as to preserve reflection amplitudes and character (Reilly and Greene, 1976). Migration sharpens features and resolves structural complications, such as buried foci and conflicting dips, so that attribute interpretation is more meaningful.

INTERPRETATION EXAMPLES

Display of three attributes, namely, phase, weighted average frequency and polarity, of a portion of a seismic section for line A in the Southern North Sea is shown in Figure 8. Figures 9 and 10 show reflection strength and instantaneous frequency sections, respectively, for a larger portion of this section. The input data have been corrected for variation of source wavelet shape before stacking and the wavelet shape has been corrected to zero phase prior to the

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complex-trace analysis. These sections have been migrated by the wave-equation method, and black-and-white phase traces form the background of the sections, except for the phase display (Figure 8a) itself. Interpreted subsurface features are identified on Figure 16.

The phase display (Figure 8a) emphasizes continuity and angularities of weak reflections because it is insensitive to amplitude. Thus, the weak-dipping reflections which subcrop at the angular unconformity just above 0.5 sec (A) delineate this unconformity. On a conventional section these reflections are so weak that it is difficult to locate the unconformity precisely. Similarly, the unconformities at B and C are made clear by onlap, downlap, and truncation configurations. A flat spot (D) can be seen associated with the gas reservoir at the crest of the anticline at 1.2 sec.

The weighted average frequency display (Figure 8b) should be compared with the instantaneous frequency display (Figure 10). Laterally constant layer sequences, such as the top of the Danian chalk (E), tend to be characterized by patterns which aid in reflection correlation, whereas the patterns change laterally for reflectors such as unconformities (B or C). Orange patterns such as underneath the Danian chalk reflections sometimes seem to be associated with fractured zones (F) (similar patterns are sometimes associated with fracture zones in East Texas). There are also low-frequency reflections (orange) in the shadow under the gas accumulation (D).

The apparent polarity section (Figure 8c) is interesting mainly for the appearance of the reflection from the gas reservoir. The reflection from the reservoir top has negative apparent polarity (blue) and the reflection from the gas-water interface (D) has positive polarity (magenta).

The reflection strength section for North Sea line A is shown in Figure 9. Major vertical lithologic changes such as from Tertiary clastics to the chalk (E) or from Triassic clastics to Permian carbonates and evaporites (G) are generally associated with high reflection strength. The reflection strength is more constant on the upper of these (Tertiary clastics to chalk), indicating that this lithologic contrast is more constant than the lower contrast where the nature

Fig. 14. Interpretation of seismic section for Gulf of Mexico line B (shown in Figures 11 and 12).
of the subcropping Permian formations changes laterally. Lateral changes in reflection strength often mark unconformities. The Carboniferous anticline (J) has some amplitude standout.

The instantaneous frequency section for North Sea line A (Figure 10) shows distinctive reflection character associated with the Danian chalk (E) and the Rotliegendes (H), but most of the other reflections change character slowly along the bedding. Note the low-frequency reflection just below the gas reservoir (D). The block faulting of the Rotliegendes (H) is emphasized by the black-and-white phase background.

Figure 11 is a portion of a reflection strength section for line B in the Gulf of Mexico. Several prominent bright spots are evident (yellow, orange, and red colors). This line is coincident with the crest of a salt ridge and is perpendicular to line C, the section shown in Figure 13 (C and B at the top of the sections indicate the intersection). Figure 12 is a weighted average-frequency section for this line. A number of low-frequency zones (orange) are in some places associated with the bright spots seen in Figure 11, and at other places the low-frequency zones and bright spots are not coincident.

The locations of two wells and information as to productive zones in a number of other wells are shown on the migrated instantaneous phase section in Figure 14. The wells are not located on the seismic line but have been projected perpendicularly onto the line, so some projection errors result. Crossdips are small, so structural features on this migrated seismic section should be nearly correct. The faults have been interpreted from the seismic data.

Self-potential (SP) logs in the two wells are shown in Figure 14. Massive shale and interbedded sand and shale zones, interpreted from the SP logs, correlate well with attribute character, especially phase. Loss of reflections (low reflection strength and decrease in phase coherence) and increased noise (higher frequencies) characterize the supernormal pressure, massive shale zones. Circles indicate production zones which have been drilled and letters indicate their order of thickness: A, less than 20 ft thick; B, 20–50 ft thick; C, 50–150 ft thick. All the production is gas. The top of the supernormal pressure is indicated by SNP.

The productive zones generally correlate well with both reflection strength and low-frequency zones, although a few productive zones do not show as obvious anomalies and a few anomalies are not associated with established production. The field is still under development and additional production may be established.

Figure 13 shows a portion of a section perpendicular to the section shown in Figures 11 and 12. Interpretation of this section is shown in Figure 15. No drilling has been carried out on the left half of this section, but several productive zones have been predicted. Other examples of the geologic interpretation of attribute measurements are given in Taner and Sheriff (1977).

CONCLUSIONS

Analysis of seismic traces as part of complex (analytic) signals allows the ready determination of the amplitude of the envelope (reflection strength), instantaneous phase, and instantaneous frequency. Color-encoded displays of attribute values aid in interpretation of seismic data relevant to stratigraphy and sometimes to hydrocarbon accumulations. The reflection-strength portrays reflectivity and hence information about impedance contrasts. The instantaneous phase emphasizes coherency and changes in dip of successive reflections. The instantaneous frequency is useful in correlation and sometimes appears to indicate hydrocarbon accumulations. Weighted average frequency aids in identification of major frequency variations, and apparent polarity sometimes helps in identifying gas accumulations. Lateral variations in all displays help localize stratigraphic changes.

ACKNOWLEDGMENTS

A number of people contributed to the work discussed in this paper. Appreciation is especially expressed to N. A. Anstey, R. O’Doherty, and others.
in Seiscom Delta, the aggregate of whose contributions have resulted in the development of these techniques.

N. A. Anstey was the pioneer in both the development of the techniques and in appreciating their geological significance. He authored two privately published booklets, Seiscom '72 and Seiscom '73, which have been important references.

The assistance of clients who elect to remain anonymous is also acknowledged, especially for their permission to publish the sections.

REFERENCES


APPENDIX A

PROCESSING APPLICATIONS OF COMPLEX TRACE

Let us define a seismic trace \( f(t) \) as the real part of analytic trace \( F(t) \), where the quadrature trace is \( f^*(t) \)

\[
F(t) = f(t) + jf^*(t)
\]

\[
= A(t) [\cos \theta(t) + j \sin \theta(t)].
\]  

\[
(A-1)
\]

Cross-correlation

The cross-correlation of two analytic traces, \( F_1(t) \) and \( F_2(t) \), is

\[
\phi(\tau) = \int_{-\infty}^{\infty} F_1(t) F_2^*(t + \tau) dt,
\]  

\[
(A-2)
\]

where the bar indicates the complex conjugate.

\[
\phi(\tau) = \int_{-\infty}^{\infty} \left[ f_1(t) f_2^*(t + \tau) + f_1^*(t) f_2(t + \tau) \right] dt
\]

\[
+ j \int_{-\infty}^{\infty} \left[ f_1^*(t) f_2^*(t + \tau) \right] dt.
\]  

\[
(A-3)
\]

Arrival time measurement

Phase measurement determines the relative arrival times of signals of similar form. This has important implications in velocity spectra analysis, velocity and dip determination, static time corrections, linear modeling, and in other processing.

As an example of a timing measurement, let us take \( f(t) \) as a 25-Hz Ricker wavelet sampled at
4-msec intervals, and with a maximum value of 1 at 4 = -1 msec. To find \( f^*(t) \) from the sampled values, we use a 22-point operator designed for the sampling interval of 4 msec. Values of \( f, f^*, \) and \( \theta \) at the three sample points closest to the maximum of \( f \) are

<table>
<thead>
<tr>
<th>( t )</th>
<th>( f(t) )</th>
<th>( f^*(t) )</th>
<th>( \theta(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4 msec</td>
<td>0.84096</td>
<td>-0.49098</td>
<td>-30.278 degrees</td>
</tr>
<tr>
<td>0 msec</td>
<td>0.98159</td>
<td>0.17489</td>
<td>10.102 degrees</td>
</tr>
<tr>
<td>4 msec</td>
<td>0.59274</td>
<td>0.71554</td>
<td>50.362 degrees</td>
</tr>
</tbody>
</table>

We estimate \( t_{\text{max}} \), the time where \( f(t) \) has its maximum value, in two ways:
1) the time where an interpolating quadratic for \( f(t) \) has a maximum; this gives \( t_{\text{max}} = -0.938 \) msec; and
2) the time where \( \theta(t) = 0 \) by linear interpolation; this gives \( t_{\text{max}} = -1.007 \) msec, which is in error by only 7 \( \mu \)sec.

**Conjugate of a convolution**

If we let \( f(t) = \int_{-\infty}^{\infty} g(\tau) s(t-\tau) d\tau \), the quadrature trace \( f^*(t) \) is given by either of the equivalent formulas

\[
f^*(t) = \int_{-\infty}^{\infty} g(\tau) s^*(t-\tau) d\tau,
\]
or

\[
f^*(t) = \int_{-\infty}^{\infty} g^*(\tau) s(t-\tau) d\tau.
\]

When \( g(t) \) is a spike sequence and \( s(t) \) is a wavelet, the natural formula to use is the first of these.

**Sum of time series**

We can consider simple filtering as a summation and measure its performance by measuring the output-to-input power ratio. The sum is given by summing real and imaginary parts,

\[
\sum_{k=1}^{N} F_k = \sum_{k=1}^{N} f_k + i \sum_{k=1}^{N} f_k^*.
\]

Power is given by

\[
P_{\text{out}}(t) = F_k(t) F_k^*(t).
\]

Hence the output-to-input power ratio is:

\[
\frac{P_{\text{out}}}{P_{\text{in}}} = \frac{\left( \sum_{k=1}^{N} F_k \right) \left( \sum_{k=1}^{N} F_k^* \right)}{N \cdot \sum_{k=1}^{N} (F_k F_k^*)}.
\]

Equation (A-8) can be used for coherence computations such as those involved in velocity analysis. The effectiveness of trace summation (stacking) can be computed on a sample-by-sample basis, eliminating the necessity of averaging over a time window.

**Product of time series**

The product of two time series is

\[
F_1 F_2 = A_1 A_2 \left[ \cos(\theta_1 + \theta_2) + j \sin(\theta_1 + \theta_2) \right].
\]

Similarly,

\[
F_1 F_2 = A_1 A_2 \left[ \cos(\theta_1 - \theta_2) + j \sin(\theta_1 - \theta_2) \right].
\]

If \( \theta_1 = \theta_2, F_1 F_2 \) will be real, but if \( |\theta_1 - \theta_2| = \pi/2 \), \( F_1 F_2 \) will be imaginary. Consequently, we can deduce the phase differences between complex time series by noting the ratio between imaginary and real parts.

If \( F_1 \) and \( F_2 \) are the same except for a phase shift of \( \theta \),

\[
F_1(t) F_2^*(t) = A^2 (\cos \theta + j \sin \theta),
\]

and the argument of the product has the constant value \( \theta \).

If \( F_1 \) and \( F_2 \) are conjugate pairs, then

\[
F_1(t) F_2(t) = -j A^2 (\cos \theta(t) \sin \theta(t),
\]

and \( \arg(F_1 F_2) = \pi/2 \), the constant phase difference between a trace and its conjugate.

**Correlation**

Let us consider a complex cross-correlation over the time window \( T \)

\[
\phi(\tau) = \sum_{t} F_1(t) F_2(t + \tau).
\]

From equation (A-3), we can write this in the following form.

\[
\phi(\tau) = \sum_{T} \left[ f_1(t) f_2(t + \tau) + f_1^*(t) f_2^*(t + \tau) \right] + j \sum_{T} \left[ f_1^*(t) f_2(t + \tau) \right]
\]
Thus, complex cross-correlation is composed of four cross-correlations which can be computed in a normal manner. If both traces are identical, then the cross-correlation function is real at zero lag. Cross-correlation can also be expressed in polar form

\[
\phi(\tau) = \sum_{\tau} A_1(t) A_2(t + \tau) \cdot \left( \cos[\theta_1(t) - \theta_2(t + \tau)] + j \sin[\theta_1(t) - \theta_2(t + \tau)] \right). \tag{A-11}
\]

**Semblance**

In the product \( F_1 \overline{F_2} = A_1 A_2 [\cos(\theta_1 - \theta_2) + j \sin(\theta_1 - \theta_2)] \), we can consider that the real part consists of the product of the modulus of one of the components with the projection of the other onto it (Figure A-1). Similarly, the imaginary part is the product of the modulus of the one with the vector component of the other which is 90 degrees out of phase.

In rectangular coordinates, we can write the product in the form

\[
F_1 \overline{F_2} = (f_1 f_2 + f_1^* f_2^*) + j(f_1^* f_2 - f_1 f_2^*),
\]

and we can now show the same proportions by dividing real and imaginary parts by the modulus

\[
F_1 \overline{F_2} = \left| F_1 F_2 \right| \left[ \frac{(f_1 f_2 + f_1^* f_2^*)}{(f_1^2 + f_1^* f_2^2)^{1/2}(f_2^2 + f_2^* f_2^2)^{1/2}} \right] + j \left[ \frac{(f_1^* f_2 - f_1 f_2^*)}{(f_1^2 + f_1^* f_2^2)^{1/2}(f_2^2 + f_2^* f_2^2)^{1/2}} \right]. \tag{A-12}
\]

We can also show that

\[
F_2 \overline{F_1} = (f_2 f_1 + f_2^* f_1^*) - j(f_2^* f_1 - f_2 f_1^*);
\]

therefore, \( F_1 \overline{F_2} + F_2 \overline{F_1} = (f_1 f_2 + f_1^* f_2^*) \), which is real. Note also that \( F_1 \overline{F_2} = (F_1 - F_2) \). Consequently, if we compute the sum of all possible pairwise cross-products between \( N \) complex numbers, the result will be real

\[
\sum_{m=1}^{N} \sum_{n=1}^{N} \left( F_m \overline{F_n} \right) = \sum_{m=1}^{N} \sum_{n=1}^{N} \left( f_m f_n + f_m^* f_n^* \right). \tag{A-13}
\]
Therefore, the average of the in-phase portion \( \phi_A \) is

\[
\phi_A = \frac{2}{N(N - 1)} \cdot \sum_{m=1}^{N-1} \sum_{k=m+1}^{N} \frac{(f_m f_k + f_m^* f_k^*)}{[(f_m^2 + f_m^* 2)(f_k^2 + f_k^* 2)]^{1/2}}.
\]

(A-14)

\( \phi_A \) corresponds to averaging the cross-correlation coefficients between real-valued time series. Note that this equation is for one sample out of each complex time series.

Equation (A-8) expressed the ratio between input and output power computed by summing \( N \) traces. Let us consider the terms in the numerator, which are squares of sums of real and imaginary parts of a trace. We know that

\[
\sum_{k=1}^{N} f_k^2 = \sum_{k=1}^{N} f_k^2 + 2 \sum_{k=1}^{N-1} \sum_{m=k+1}^{N} f_k f_m,
\]

and

\[
\left( \sum_{k=1}^{N} f_k^* \right)^2 = \sum_{k=1}^{N} f_k^* 2 + 2 \sum_{k=1}^{N-1} \sum_{m=k+1}^{N} f_k^* f_m^*.
\]

Semblance is defined as the power of the sum divided by the average power of the components of the sum (Taner and Koehler, 1969; Sheriff, 1973). Consequently we can compute the semblance coefficient \( \sigma \) for a complex time series as

\[
\sigma = \frac{\left( \sum_{k=1}^{N} f_k \right)^2 + \left( \sum_{k=1}^{N} f_k^* \right)^2 - \sum_{k=1}^{N} (f_k^2 + f_k^* 2)}{(N - 1) \sum_{k=1}^{N} (f_k^2 + f_k^* 2)},
\]

(A-15)

where

\[
-1 \leq \sigma \leq 1.0.
\]

**APPENDIX B**

**COMPLEX TRACE EXAMPLE OF RICKER WAVELET**

Let \( C(\omega) = (2/\pi)^{1/2} \omega^2 e^{-\omega^2/2}, \phi(\omega) = 0 \) in equation (11); this defines a Ricker wavelet. Then

\[
f(t) = (2/\pi)^{1/2} \int_0^\infty \omega^2 e^{-\omega^2/2} \cos \omega t d\omega
\]

\[
= (1 - t^2) e^{-t^2/2}.
\]

The constant factor \((2/\pi)^{1/2}\) in \( B(\omega) \) was chosen so that \( f(0) = 1 \). The conjugate trace is given by

\[
F^*(t) = \int_0^\infty \omega^2 e^{-\omega^2/2} \sin \omega t d\omega = 2(2/\pi)^{1/2} t
\]

\[= \sum_{m=1}^{\infty} \frac{(2m - 3)}{(2m + 1)!} t^{2m+1} e^{-t^2/2},
\]

where \((2m - 3) = 1 \) for \( m = 1 \), \( 1.3 \ldots (2m - 3) \) for \( m \geq 2 \).

The maximum value of \( B(\omega) \) is attained for \( \omega = \sqrt{2} \) radians per unit time. By a suitable choice of the unit of time, this maximizing value of \( \omega \) can be made equal to any desired frequency. If we take the unit of time as \( 50(2)^{-1/2} \) sec, the maximizing frequency is 25 Hz. Graphs for such a Ricker wavelet are shown in Figure 5 and data are listed in Table 1. (Since the wavelet is symmetrical, only half of the wavelet is listed.)