An Electrical Rock Physics Model for Partially Interconnected Fluid Inclusions/Cracks
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Summary
A rock physics model is presented to calculate the effective resistivity of a rock with partially interconnected fluid inclusions or cracks. The model uses the differential effective medium models and the probability of interconnection between inclusions. The level of interconnection between the fluid inclusion is controlled by the volume fraction of the fluid and the aspect ratio of the inclusions. The model is independent of inclusion size and assumes a statistically random distribution of inclusions.

Introduction
Electrical resistivity is an important physical property measurement for reservoir characterization. Resistivity can be measured in a variety of ways. It can be logged at the borehole, inverted from controlled source electromagnetic (CSEM) surveying and measured in the laboratory. Each of these methods measure resistivity in different ways and at different length scales. Electrical resistivity is principally used to determine the fluid saturation of reservoirs. Resistivity measurements are also used to develop CSEM feasibility studies and starting models for CSEM inversion. Robust rock physics models are required in order to achieve these objectives. Empirical models have often been used however these models require large data sets to fit the model’s empirical parameters too. Theoretical rock physics models are more powerful because they can be adapted to the geology/microstructure of the rock.

The physical properties of sedimentary rocks such as electrical resistivity are strongly dependent on the shape, distribution and interconnection of conducting fluid inclusions. Rock physics models use these parameters to predict the physical properties in order to interpret geophysical data. In sediments such as clean sands this is relatively easy and Archie’s equation (Archie 1942) has long been used to achieve this. In rocks such as shale (figure 1) this is more difficult. Shale is composed of a range of components including high volumes of clay which, along with the fluid, add conductivity. The components may be either randomly distributed, fully aligned or some other intermediate distribution causing anisotropy. Shales also tend to be highly fractured which further alters the bulk resistivity on a range of length scales. Rock which contain conductive material such as clays must be modeled using rock physics models in which all of the constituents are permitted conductivity. Models such as Archie’s equation do not permit any of the solid phases to be conductive. To build a rock physical model to account for fractures like inclusions in an effective medium, the degree of fracturing, the alignment of the fractures and the interconnection of the fractures must be taken into account.

In this study a rock physics model is developed to determine the degree of interconnection between idealized fluid inclusions. The method uses a statistical approach to calculate the number of inclusions in a host medium that overlap a single inclusion. This degree of interconnection is controlled by the aspect ratio and the volume fraction of the inclusions. The inclusions are assumed to be oblate spheroids and their aspect ratio is allowed to alter so that spherical to crack-like pores can be modeled. The method also allows the inclusions to be distributed in various geometries from fully random to fully aligned. The degree of interconnection can then be applied to electrical effective medium models, such as the Hashin-Shtrikman or Differential Effective Medium (DEM) models, to navigate between upper and lower bounds to determine the electrical resistivity of a partially connected medium. Using models such as the DEM allows both the vertical and horizontal resistivities to be calculated and therefore anisotropy to be determined. It also allows the matrix some conductivity, important when modeling sediments with significant amounts of clay.

Figure 1: SEM of shale showing micro fracturing (Han 2010).
Inclusion Interconnection

The degree of inclusion interconnection is important when modeling effective resistivity. If fluid inclusions are interconnected the resistivity will be significantly lower than when the inclusions are isolated. Therefore, we need to calculate the probability of connection between inclusions.

Inclusion A (I_A) is located within an arbitrary volume and around I_A is an area of influence. If another inclusion’s center falls within this area it may intersect I_A. If it falls outside the area of influence, it will not intersect I_A. To calculate the number of inclusions that fall within the area of influence (i) we use the following equation

\[ i = \left( \frac{V_i}{V_s} \right)^N, \]

where N is the total number of inclusions, V_i is the volume of the area of influence and V_s is the total volume of the arbitrary volume. V_s is determined by semi-axis lengths of the inclusions (Figure 2) and is given by,

\[ V_s = \frac{4}{3}(a + a)(b + b)(c + a)\pi, \]

where a and b are the long semi-axis of the inclusion (assuming that the inclusion is oblate) and c is the short semi-axis.

To calculate the initial probability of inclusion connection (P_i) we can use Schmelling (1983) probability of interconnection equation, however, if the inclusions are randomly orientated, inclusion I_B may not always intersect with I_A even if it’s center is located within the area of influence. Therefore, the probability of connection must be adjusted to account for this. Consider just 2 inclusions, I_A and I_B. It is assumed that these inclusions are the same shape and size. I_A stays fixed in space \([x=0, y=0, z=0]\) and does not rotate \([\theta=0, \psi=0, \phi=0]\). I_B is placed at some other point and we determine whether the two ellipsoids intersect.

Determining whether two inclusions intersect mathematically can be difficult. It is assumed that the inclusions are ellipsoids and an ellipsoid can be described by matrix equations. I_A is given by:

\[ XAX^T = 0, \]

and I_B by

\[ XBX^T = 0, \]

where

\[ X = [x \ y \ z \ 1] \]

and

\[ S = 1/2 \begin{pmatrix} 2A & D & F & G \\ D & 2B & E & H \\ F & E & 2C & J \\ G & H & J & 2K \end{pmatrix}. \]

If any X exists that satisfies both equations then I_A and I_B intersect at that point. Determining X however is difficult. One possible way is to evaluate every possible point on I_A to see if it satisfies I_B. However, this method is time consuming and unreliable especially for ellipsoids that only just touch. Therefore, the method of Alfano and Greer (2003) has been used. This method involves adding an extra dimension to the solution space and examining the eigenvalues that are associated with degenerate quadric surfaces. Essentially the eigenvalues of \(A^{-1}B\) are calculated. If the eigenvalues are equal the ellipsoids intersect and if they are not then they don’t. This is much more reliable method and far less time consuming. I_B is rotated in all directions (assuming random orientation of the inclusions) to determine what percentage of positions the two inclusions intersect.

Once the percentage of interconnections has been calculated for this single I_B position, all other I_B positions must be calculated as well to give the final percentage of connections (C). To achieve reliable results the sampling must be even over the whole area of influence, \(dx = dy = dz\).

The percentage of connections (C) can now be applied with the number of i centers within the area of influence to the
Schmelling (1983)’s equation to give the final probability of inclusion connection ($P_c$).

$$P_c(a, \beta) = \begin{cases} \frac{l_c(a, \beta)}{i_{\text{max}}} & l_c \leq i_{\text{max}} \\ 1 & l_c \geq i_{\text{max}} \end{cases}$$

where,

$$l_c = \left(N \frac{V_f}{V_f^c}\right) \text{C}.$$ 

and $i_{\text{max}}$ is the maximum number of inclusions that $I_A$ has to be connected to, for the medium to be considered interconnected. The value of $i_{\text{max}}$ therefore is critical. Schmelling (1983) determines that an appropriate value for $i_{\text{max}}$ should lie between 4 and 10. O’Connell & Budiansky (1977) give crack densities with corresponding values of $i$ of between 2 and 4. Therefore Schmelling (1986) uses $i_{\text{max}} = 4$ for a totally interconnected fluid. Figure 3 shows the $P_c$ as a function of aspect ratio and inclusion volume fraction.

**Bound Navigation**

Once the probability of connection between inclusions has been calculated it can be used to calculate the effective resistivity of a partially interconnected medium. The method starts with two bounds calculated using the DEM method (Bruggeman, 1935; Sen et al 1981; Berryman, 1995). The lower (conductive) bound assumes that the fluid is fully connected at all porosities (fluid starting medium).

The upper (resistive) bound is calculated assuming the matrix (solid grains) is connected at all volume fractions. Greer’s 2001 method is used to navigate between the two bounds using $P_c$ to calculate the effective resistivity of a partially interconnected medium ($\rho_{\text{inter}}$).

$$\frac{1}{\rho_{\text{inter}}} = \sigma_{\text{inter}} = \sigma_{\text{DEM,fluid}}^{P_c} \times \sigma_{\text{DEM,matrix}}^{(1-P_c)}.$$ 

$\sigma_{\text{DEM,fluid}}$ is the conductivity of the lower (conductive) bound and $\sigma_{\text{DEM,matrix}}$ is the conductivity of the upper (resistive) bound. Figure 4 (left) shows the final result when $i_{\text{max}} = 4$ and the aspect ratio of the inclusions is 0.25. When the volume fraction of the inclusions is zero the $\rho_{\text{inter}}$ lies on the resistive bound. As the inclusion volume fraction increases the inclusions start to become interconnected and $\rho_{\text{inter}}$ moves away from the resistive bound towards the conductive bound. When the inclusion volume fraction is approximately 0.35 the inclusions are considered fully interconnected and $\rho_{\text{inter}}$ follows the conductive bound. Figure 4 (right) shows the effect of increasing $i_{\text{max}}$. The larger the value of $i_{\text{max}}$ the larger the inclusion volume fraction has to be before the model approaches the conductive bound.

**Conclusions and Discussion**

A rock physics model has been developed for partially interconnected randomly orientated inclusions. This model has the advantage that it is a theoretical model without empirical contents and therefore does not need a large data

Figure 3: Final probability of inclusion connection when $i_{\text{max}} = 4$ (left) and $i_{\text{max}} = 10$ (right). 1 on the colourbar represents 100% probability and 0 indicates 0 %.
Electrical Rock Physics Model for Fractures

set to calibrate it. It uses the DEM model, which allows the generally resistive matrix some conductivity (unlike models such as Archie’s). This means that it could be used to model sediments that contain significant volumes of clay. Unlike the traditional DEM model however it allows both phases to be partially interconnected. This allows a more realistic medium to be modeled. The probability of inclusion connections could also be for other rock physics modelling such as elastic and permeability modeling where the degree of connection between pores is also important.

Developing rock physics models which can model the structure of the sediment is important as they improve fluid saturation calculations, fluid substitution modeling, and allow resistivity to be calculated in areas where resistivity data is missing. This is especially important when conducting CSEM modeling and data interpretation. Poor rock physics models may lead to poor feasibility studies being conducted and consequently bad survey design. Incorrect resistivities may also lead to poor background models being used when inverting CSEM data. Finally, good rock models which can model the all sediments (not just sandstone) lead to an improve interpretation of the entire sedimentary column. Again this is important for CSEM as the entire sedimentary column must be modeled not just the reservoir.

This type of rock physics modeling is also important when conducting joint seismic and electrical interpretation of seismic and CSEM surveys. In almost all geophysical surveying there is a degree of non-uniqueness. By jointly interpreting both elastic and electrical measurements this non-uniqueness can be reduced. Equivalent elastic and electrical rock physics models can be developed in which the input parameters (such as aspect ratio, composition etc.) are the same. These models can then provide the link between seismic and CSEM surveying.

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Figure 4. Effective partially interconnection resistivity using DEM conductive and resistive bound for an aspect ratio 0.25 and two different $i_{\text{max}}$ values (4 and 10).
EDITED REFERENCES

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REFERENCES


