Joint Time/Frequency Analysis, Q Quality factor and Dispersion computation using Gabor-Morlet wavelets or Gabor-Morlet transform

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Introduction:

Absorption, dispersion and the related Q quality factor are one of the more important seismically measurable factors that relate to porosity and rock physics. Unfortunately, most of the previous methods contain high degrees of uncertainty. This is due to the very subtle change of seismic data characteristics over the measured distance. However, robust computation of these parameters will greatly improve our ability to estimate the reservoir characteristics. The purpose of this paper is to discuss one method to calculate the attenuation, Q and the dispersion values from the instantaneous spectra. Instantaneous spectra can be obtained by the Wigner transform, or by the Gabor transform (Gabor, 1946), which we will discuss in this report.

I would like to point out that Dr. Morlet and his associates introduced Gabor's work to the geophysical industry. He modified Gabor's subdivision of the frequency domain that retained the wavelet shape over equal octave intervals. This is now called the Gabor-Morlet transform. This is also recognized as the first indication of generalized wavelet transform. I have included references to a number of papers by Dr. Morlet. Upon reading Morlet's paper, Dr. Koehler became very impressed with the idea, which resulted in a number of theoretical and practical papers. The application presented here is one of the results.

Method:

The commonly used method is the conventional spectral division. We will form this division on the Gabor-Morlet decomposed data rather than in the Fourier domain. By definition, absorption relates to the energy loss per cycle and dispersion relates to propagation velocity varying as function of frequency. The energy loss effects the amplitude spectra of the wavelets. Waves going through any medium loose some of their energy by conversion to heat or by plastic deformation, hence the spectrum of a transmitted wavelet will contain less energy than the incident one. If the propagation velocity is constant for all frequencies, this loss relates to a percentage of the energy loss per cycle. Since the same distance will be traversed by more cycles of higher frequencies, than the low frequencies (longer wavelength), the higher frequencies will naturally suffer more losses than the lower frequencies, but the phase spectrum will remain the same. In a medium where there is dispersion and energy loss,, both the amplitude and the phase spectra will change according to the characteristics of that medium. It is interesting to note that pressure and shear waves will have different dispersion and attenuation characteristics, which are effectively used for characterization.

Theory:

We assume the constant $\boldsymbol{Q}$ condition, that is the energy loss relates to the number of cycles over a travel distance spanned. In this case, the original amplitude spectrum of the seismic wavelet $A_0(f)$ will be changed to;

$$\left|A_t(f)\right| = \left|A_0(f)\right| . exp(-\pi f t / Q) \qquad \textbf{(1)}$$

where $\boldsymbol{t}$ is the travel time from origin to the target. The Q is estimated from the ratio of the amplitude spectra of the wavelets obtained above and below the area of interest;

$$ln\left|\frac{A_2(\,f\,)}{A_1(\,f\,)}\right| = -pf\,(\,t_2 - t_1\,)\,/\,Q \qquad \textbf{(2)}$$

The main problem stems from the zero or near zero values of $A_1(\,f\,)$ which give rise to unusable values and large estimate errors. However, over coherent zones the ratio gives estimates that are more accurate. To take advantage of this we use the amplitude of the spectra as weights in least square line fitting for Q estimation. The spectral ratio's problem with zeros on the unit circle are due to computational inaccuracies of autocorrelation functions or to the effects of various forms of noise, reflectivity series and the man made effects of notch filters. Since spectral division is the same as the z polynomial division, we can obtain the desired results by dividing two stable polynomials. These stable polynomials are conventionally computed by unit-step prediction error, better known as spiking operators. These operators are the minimum phase inverse of the minimum phase equivalent of the seismic wavelet of a particular computation zone. Since these operators are minimum phase, then, they can be inverted or used in polynomial division without any instability. These inverse wavelets can be computed from autocorrelation functions by the familiar Wiener-Levinson algorithms.

Gabor -Morlet Transform Method:

The Gabor-Morlet transform is performed by filtering the seismic data by a series of Gabor-Morlet wavelets. The results are narrow-band analytic traces. The amplitude and phase of each narrow band filtered output represents the average amplitude and phase of the narrow-band part of the input trace.

The proposed method includes the computation of the analytic traces from the original input. A user selected number (N) of -Gabor-Morlet wavelets are convolved with the data to produce N sub-band analytic traces. These sub-band traces are normalized by dividing them by the original trace envelope. This will remove the amplitude variation of individual reflected events, leaving only the variations between the individual sub-band traces. These trace amplitudes can be displayed as instantaneous amplitude spectra of the input trace. Similarly, joint time-frequency phase spectra are generated as the arc-tangent of the imaginary to real parts of each sub-band. These are displayed as the instantaneous time-frequency phase spectra.

The envelope peaks of the input trace correspond to the time where all of the sub-band components are in-phase. If we pick envelope and phase values of each sub-band, we will have the specific amplitude and phase spectra content of the input wavelet. Absorption and dispersion estimates are then obtained from the differences of log amplitude and phase between adjacent wavelets in the time direction. We will cover the detail of the computation below. Gabor wavelet theory is reviewed in an excellent paper by Koehler (1983). Here, I will describe the practical application of the Gabor-Morlet wavelet theory.

Gabor-Morlet Wavelet Specifications

Time domain response of the wavelet;

$$g_j(\,t\,) = exp(-a_j t^2\,).exp(\,iw_j t\,): \qquad \textbf{(3)}$$

Corresponding frequency domain response is;

$$G_j(w\,) = \int_{-\infty}^{\infty} g_j(\,t\,)exp(\,iw_j t\,)dt = \sqrt{\frac{p}{a_j}}.exp\{-(w - w_j\,)^2\,/\,4a_j\,\} \qquad \textbf{(4)}$$

$\Delta t_j = k_1\,/\,w_j$ width of j'th wavelet in time domain, **(5)**

$\Delta w_j = k_2 w_j$ width of j'th wavelet in frequency domain, **(6)**

where;

$\Delta t_j \Delta w_j = k_1 k_{2j}$ constant **,** **(7)**

The "width" of a function is defined as the interval between which the function is equal to or more than one-half its maximum value; i.e.,

$$exp\{-a_j(\Delta t_j/2)^2\} = 1/2 \qquad \text{and,} \tag{8}$$

$$exp\{-(\Delta w_j/2)^2/4a_j\} = 1/2. \tag{9}$$

From equations 6 and 7 we get,

$$a_j(\Delta t_j)^2/4 = ln(2) \qquad \text{and}$$

$$(\Delta w_j)^2/16a_j = ln(2).$$

We compute ;

$$(\Delta t_j)^2(\Delta w_j)^2 = 64.(ln2)^2 \qquad \text{this will result in;}$$

$$\Delta t_j \Delta w_j = 8\,ln2 \tag{10}$$

If we choose $k_1 = 4p$ and $\Delta t_j = 4p/w_j$, this makes the wavelet amplitude equal to one-half of its maximum at an interval of one period on each side of the maximum point. From equations 7 and 10 we get;

$$k_2 = \frac{2}{p}ln(2) \qquad \text{and} \qquad \Delta w_j = \frac{2\,ln2}{p}w_j. \tag{11}$$

The value of a_j is determined from equation 8 or 9;

$$a_j = \frac{ln2}{4p^2}w_j^2.$$

User specifies the usable frequency band for spectra computation. Then, this band is subdivided into equal intervals in octave representation of the frequency axis.

Figure 1 shows the results of the decomposition. We have designed 17 Gabor-Morlet sub-band complex filters. Real and imaginary parts of the sub-band data was generated by convolving the input data with corresponding filters. Amplitude spectra is generated in the conventional way as the square root of the sum of squares of the real and imaginary parts of the sub-band traces. The phase spectrum is the arc-tangent of the ratio of imaginary to the real part of the sub-band trace. Figure 1A shows the input data taken from a 3-D stacked and migrated data set. Figure 1B is the Joint Time/Frequency Analysis amplitude spectra. Figure 1C is the phase spectra. On both of the displays, the horizontal axis is the frequency and the vertical axis is the time. The seismic data shows that there is a zone of thin-bedded sequences between 500 milliseconds and 1400 milliseconds. Amplitude spectra in this zone are wider band. Limits of this zone marked by low amplitude areas. Events below 1500 milliseconds have different character, widely apart and separated by events of low reflectivity. This also has shown on the amplitude spectra. Phase spectra is not influenced by the amplitude of traces, thus is appears with uniform scale. Colors represent the phase angle.

In Q computation, we need to compute the amplitude spectra ratio of two adjacent events. Joint Time/Frequency analysis provide us the spectra of all events on the seismic trace. We can compute the log of amplitude ratio between any two events. Since we are interested in the slope of this ratio, amplitude differences of two events will not adversely affect our computation. It may be necessary to run the analysis with several wide band decomposition to establish the frequency band over which more reliable results may be obtained. The noise in high frequencies will give erroneous results. Tuning thickness may result in peaks at various frequency bands. Once the usable bandwidth is established, a section showing the interval Q estimates is generated. These values can also be used for lithological classification.

The phase spectra will provide information for dispersion estimation. Attributes picked at the peak of the envelope represent the average of the wavelet attribute. That is why we pick the amplitude spectrum at the time of envelope peak for Q computation. Phase spectra is picked the same way. If we look at the figure 1C, we observe that most of the spectra of the events are horizontal, which means that these wavelets are zero phase, and their rotation angle is the phase corresponding to the envelope peak. Therefore, computation of dispersion consists of determining the phase differences at each sub-band trace and compute an average phase delay per cycle per second. Since dispersion is related to absorption, higher levels of dispersion will point to higher levels of absorption, which may indicate fracture in carbonates or unconsolidated sands in clastic environment.

Conclusions:

In this report, I have presented Joint Time-Frequency analysis using the Gabor-Morlet decomposition. This analysis makes it possible to measure absorption or Q quality parameter and dispersion directly between two events. Time-Frequency display is a valuable tool in itself, it shows the major boundaries where considerable change of Q and/or dispersion.

There is an excellent article by Qian and Chen (1999) in IEEE Signal Processing magazine. This article contains many good references relating the Joint Time-Frequency Analysis. This issue (March 1999) of Signal Processing magazine contains several other articles on the application of Gabor expansion.

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Figure 1. Gabor-Morlet decomposition